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THE USE OF LEARNING CURVES

IN FINANCIAL ACCOUNTING

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College of Commerce and Business Administration

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Abstract

The Use of Learning Curves in Financial Accounting

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Despite their widespread use in other disciplines, there have been few applications of learning curve models in financial accounting. However, recent developments indicate that there are a number of significant financial accounting applications of such models. These applications include forecasting, cost allocation, and human resource accounting. This article presents a typical model based on the learning curve phenomenon, discusses its underlying assumptions and limitations, and indicates how it might be used in financial accounting. Special attention is given to the steps an auditor would have to take to satisfy himself that the use of such a model in financial accounting is appropriate and correct.

THE USE OF LEARNING CURVES IN FINANCIAL ACCOUNTING

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When variable production costs decline in a systematic manner as the number of units produced increases, the production process is said to be following the learning curve phenomenon. For many years accountants in industry and the management services departments of accounting firms have used models based on this phenomenon for such diverse purposes as contract bidding, production scheduling, variance analysis, and financial planning.

Until recently financial accountants have had little need to be familiar with learning curve models. However, there now appear to be a number of financial accounting applications of these models. These applications include statement forecasting, cost allocation, and human resource accounting. Because of these potential uses, financial accountants may find themselves using or verifying the use of such models in the future.

This article is divided into two parts. In the first part a typical model based on the learning curve phemonenon is presented. In the second part, examples of the potential financial accounting applications of that model are described.

LEARNING CURVE MODEL

Background

T. P. Wright is credited with formalizing the first learning curve model. After observing aircraft production for some time, he found a constant decrease in the cumulative average production time as output doubled. By studying previous production records he was able to determine the rate of decrease in production times for similar kinds of aircraft. Determining the rate of decrease in production time made it possible for him to predict production times and delivery schedules for future aircraft with a high degree of accuracy.[1]

In 1943, F. J. Montgomery reported on a study he made of the construction of liberty ships. He noted that between December, 1941, when the first two ships were delivered, and the end of April, 1943, the average man-hour requirements per vessel delivered was reduced by more than one half. Montgomery was one of the first to realize the wide applicability of learning curve models when he concluded that a study of the production figures of any company manufacturing a complex but standardized item would probably reveal a similar trend.[2]

Since becoming formalized, learning curve models have been used in industries as diverse as airframe assembly, electronic products, home appliances, shipbuilding, textiles, and defense. There also appear to be significant potential applications in residential home construction and computer assembly.

The foundation of all learning curve models is the belief that employees learn as they work. The more often they repeat an operation the more efficient they become. The result is declining unit production times and costs. It should be noted that the "employees" referred to above includes managerial as well as production personnel. While production workers become more skilled in performing their assigned tasks, management becomes more skilled in organizing the various factors of production.

In the literature some authors have preferred to use the terms "progress curve," "time reduction curve," "improvement curve," or "experience curve," rather than "learning curve," because of their belief that a pure learning curve should reflect only the rate of the production workers' learning. The term "learning curve," as used in this article, is intended to be a broad concept that includes both the increased productivity of production workers and the improved organization of the factors of production by management.

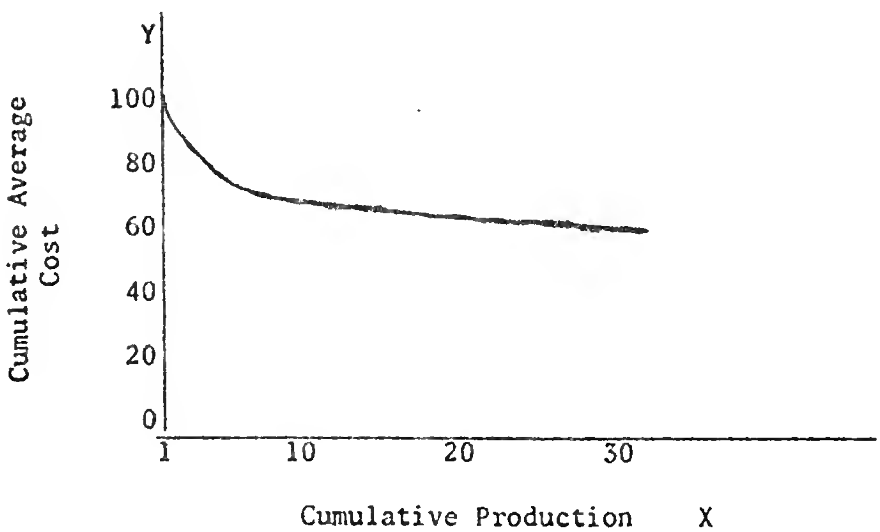
The Model

A widely adopted model based on the learning curve phenomenon states that whenever the total quantity of units produced doubles, the cumulative average unit cost declines by a constant percent.[3] Consider the example presented in Table 1 and Figure 1. It cost \$100 to produce the first unit, \$80 to produce the second unit, and \$144 to produce both the third and fourth units. Every time the total quantity of units produced doubled the cumulative average unit cost declined by 10 percent, from \$100 to \$90 to \$81. We might also say that every time the total quantity of units produced doubled the cumulative average unit cost was 90 percent of its

TABLE 1
90 Percent Learning Curve

Units Produced	Group Cost	Group Average Cost	Cumulative Average Cost	Percent Decline
1	$100	$100	$100	--
2	80	80	90	10
3-4	144	72	81	10
5-8	259.2	64.8	72.9	10
9-16	466.4	58.3	65.6	10
17-32	844.8	52.4	59.0	10

FIGURE 1
Cumulative Average Costs--90 Percent Learning Curve



previous amount. This 90 percent, which is the compliment of the rate of decline, is used to identify this particular learning curve.

Except for common practice, there is no particular reason to identify a learning curve by the compliment of the rate of decline in cumulative average unit cost. Because this practice is followed, the higher the rate of decline the lower the learning curve percent. Conversely, the lower the rate of decline the higher the learning curve percent. With no decline in cumulative average unit cost the learning curve percent is 100.

All uses of learning curve models require that past production data be analyzed to determine the rate at which production costs decline. Then, under the assumption that the same rate of decrease that occurred in the past will continue in the future, estimates of future production costs are made. While these steps can be performed by plotting a scatter diagram on log-log paper and extrapolating, a more accurate method is to statistically fit an equation to a curve similar to that in Figure 1. That equation, or a variation of it, can then be used to estimate future production costs.

The general equation that best fits the learning curve model described above and the curve in Figure 1 is:

$$Y = a/X^b \; . \tag{1}$$

Where:

X = Cumulative productions (measured on the horizontal axis);

Y = Cumulative average cost (measured on the vertical axis);

a = Computed cost of the first unit (vertical axis intercept); and

b = Exponent which accounts for the slope of the learning curve.

The parameters (a and b) of equation 1 can be found by using logarithms to transform this exponential function into a linear one, and then applying

least squares regression analysis to the logarithms of actual values of X and Y:

$$\log Y = \log a + b\log X \quad ; \tag{2}$$

$$b = \frac{n\Sigma(\log X \log Y) - \Sigma\log X\log Y}{n\Sigma(\log X)^2 - n(\Sigma\log X)^2} \quad ; \tag{3}$$

$$\log a = \frac{\Sigma\log Y}{n} - \frac{b\Sigma\log X}{n} \quad . \tag{4}$$

Where:

n = number of pairs of X and Y values used.

These values of a and b are then used in equation 1 to estimate cumulative average unit costs at larger levels of cumulative production. The total variable cost of X units is estimated by multiplying the estimated cumulative average production cost of X units by X:

$$T = XaX^b \quad . \tag{5}$$

This can be reduced to:

$$T = aX^c \quad . \tag{6}$$

Where:

$c = 1-b$.

The variable cost of a particular unit is estimated by computing the difference in estimated total variable production cost between two successive units:

$$U = a(X^c - (X-1)^c) \quad . \tag{7}$$

If T dollars are available to cover variable production costs the total number of units that can be produced with T dollars is:

$$X = \text{antilog}(c\log T - \log a)/c) \quad . \tag{8}$$

Where:

X = number of units which can be produced with T dollars given values of a and c.

While the necessary calculations can be performed by hand the most expedient procedure is to utilize a computer for this work.[4]

Limitations and Assumptions

The mathematical limits of the learning curve model presented above are 100 percent and 50 percent. If no "learning" occurs, the cumulative average cost per unit does not change and the model follows a 100 percent learning curve. Given any level of output, the cumulative average time per unit at that level of output is the same as that at any lower level of output. If the learning curve percent were 50, the model would indicate that the second unit cost nothing to produce. Given a cost of $100 for the first unit the only way for the cumulative average cost of two units to be $50 would be for the second unit to cost nothing. Jordan believes that the mathematical properties of learning curves makes a learning curve of less than 70 percent difficult to envision.[5]

Figure 1 shows that as total production increases the learning curve soon reaches a point where the difference in production time between successive units approaches zero. When this occurs the learning curve is said to have reached a "steady state." While a production process may still be following the learning curve phenomenon, once 100,000 units have been produced with a 90 percent learning curve, an additional 100,000 units must be produced before there is an additional 10 percent decline in cumulative average cost. In any event, after a large number of units have been produced the use of learning curves to project future production costs is of little value.

Learning curve models use data on past production costs to estimate future production costs. Hence, the primary assumption of the model is that estimates of the future can be extrapolated from the past. If there is a major change in the product, the production process, personnel, or input costs, estimates based on past production costs may not be appropriate.[6]

Implementation of the model also assumes that sufficient past data is available to determine its parameters. If only a small number of observations of past production costs are used in determining these parameters, random fluctuations in production costs could lead to errors.

The basic model also assumes that all variable production costs decline at the same rate. If, for example, unit labor and materials costs decline at different rates, this assumption is not valid. Fortunately, if this problem is recognized and the appropriate data is available, it can be overcome by applying the model separately to each of these cost elements.

Finally, the model assumes that all variable unit production costs to which the model is applied decline in accordance with the learning curve phenomenon. If they do not, estimates of future production costs can be materially in error.[7]

FINANCIAL ACCOUNTING APPLICATIONS

Despite the widespread use of learning curve models for planning and control, there has been little need for financial accountants to be familiar with them. Only in the aircraft industry have learning curve models been used for financial accounting purposes, and even there they have been used only in very limited circumstances.[8] There now appear to be a number of financial accounting applications of these models. In particular, learning curve models can be used for statement forecasting, cost allocation, and human resource accounting.

Forecasting

Recent issues of accounting journals have contained numerious articles on financial forecasts. In the near future auditors may be faced with the task of commenting on forecasts made by management. If management uses a learning curve model in preparing forecasts, the auditor will have to determine if the use of such a model is appropriate and if the model is used correctly. To do this, the auditor must be familiar with such models and their underlying assumptions and limitations.

Consider the following example:[9]

In 19x1 the management of the XYZ Company, after a careful analysis of anticipated costs and revenues, began production of a new line of sailboats. Sales forecasts indicated that 124 of these boats could be sold during the

next four years at a price of $10,000 each. In 19x1, XYZ built and sold 20 boats as planned.

All of XYZ's expenses are directly associated with production. Factory overhead and direct labor are each incurred at the rate of five dollars per direct labor hour. From XYZ's production records, the information in Table 2 was obtained. Based on this information and an estimate of the total number of production hours available in 19x2, the 19x1 income statement and the forecasted 19x2 income statement were prepared.

	19x1 Actual	19x2 Forecast[10]
Sales	$200,000	$300,000
Cost of Goods Sold	218,950	280,448
Net Income (loss)	$(18,950)	$ 19,552

There is little doubt that investors would be interested in obtaining information about forecasted earnings for 19x2. This is especially true given the XYZ Company's 19x1 loss of $18,950. There is also little doubt that management would be pleased to provide such a forecast. If anything, management would likely desire to provide forecasts for 19x3 and 19x4 also.

In satisfying himself that the use of a learning curve model for forecasting is appropriate and that the model is used correctly the auditor will have to answer the following types of questions:

Is the production of sailboats likely to display the learning curve phenomenon?

How many observations have been used to determine the parameters of the learning curve model?

Do these parameters explain a large proportion of the past variation in the cumulative average production time (is the coefficient of determination high)?

TABLE 2

Information From XYZ Company's Production Reports

BOAT NUMBERS	DIRECT LABOR HOURS	DIRECT LABOR COSTS (nearest dollar)	VARIABLE FACTORY OVERHEAD (nearest dollar)	DIRECT MATERIALS	TOTAL COSTS
1-4	2,890.17	$14,451	$14,451	$24,000	$52,902
5-8	2,020.26	10,101	10,101	24,000	44,202
9-12	1,784.94	8,925	8,925	24,000	41,850
13-16	1,647.48	8,237	8,237	24,000	40,474
17-20	1,552.26	7,761	7,761	24,000	39,522
					$218,950

Is it reasonable to assume that labor and overhead costs will decline at the same rate?

Do all costs or times to which the model is applied decline in accordance with the learning curve model?

Is it likely that there will be a major change in the product, production process, personnel, or input costs in the future? If so, does the model incorporate these changes or do the forecasted statements contain references to possible errors caused by them?

Have production capacity constraints been properly included in the model? Are they reasonable?

Will there be a change in the selling price of this line of sailboats?

Are the mathematical computations used in the forecast correct?

The above questions address themselves to the major assumptions and limitations underlying the use of learning curve models. After satisfying himself about them the auditor could make a statement noting that the

forecasted income statement was developed with the use of a learning curve model, and that the use of such a model was, to the best of his knowledge, both appropriate and correct.

Cost Allocation

In a recent issue of The Accounting Review, Morse presented a cost allocation model based on the learning curve phenomenon.[11] He argued that when production costs follow the learning curve phenomenon the use of a cost allocation model based on the learning curve phenomenon results in a better matching of production costs with revenues than does matching actual unit production costs with revenues.

With the use of an example, Morse showed that the use of a learning curve cost allocation model would result in accounting income measures closer to economic income, and accounting asset values that serve as better predictors of economic income than would the matching of actual unit production costs with revenues.

According to Morse:

> The learning curve (L-C) cost allocation model projects production costs over the entire anticipated life cycle of a project on the basis of cost data for the first few units of production. Using this data, comparisons are made between the projected cost of each unit and the expected average unit cost of all anticipated production. As production takes place any excess of the projected cost of each unit over the expected average cost of all anticipated production is charged to a deferred production expense account and inventory is charged with an amount equal to the expected average unit cost of all anticipated production. When the projected unit cost is less than the expected average unit cost of all anticipated production, the difference is deducted from the deferred production expence account and inventory is charged with an amount equal to the expected average unit cost of all anticipated production. As production takes place any difference between actual and projected costs are written off as a period variance unless a change in a

parameter of the model occurs. In effect, the projected unit cost becomes the standard cost.

Figure 2 shows the flow of costs under the L-C cost allocation model.

The effect of using the L-C cost allocation model is to decrease the amount of production costs charged to inventory and to the cost of goods sold in early periods, raising reported income, and to increase the amount of production cost charged to inventory and the cost of goods sold in later periods, lowering reported income.

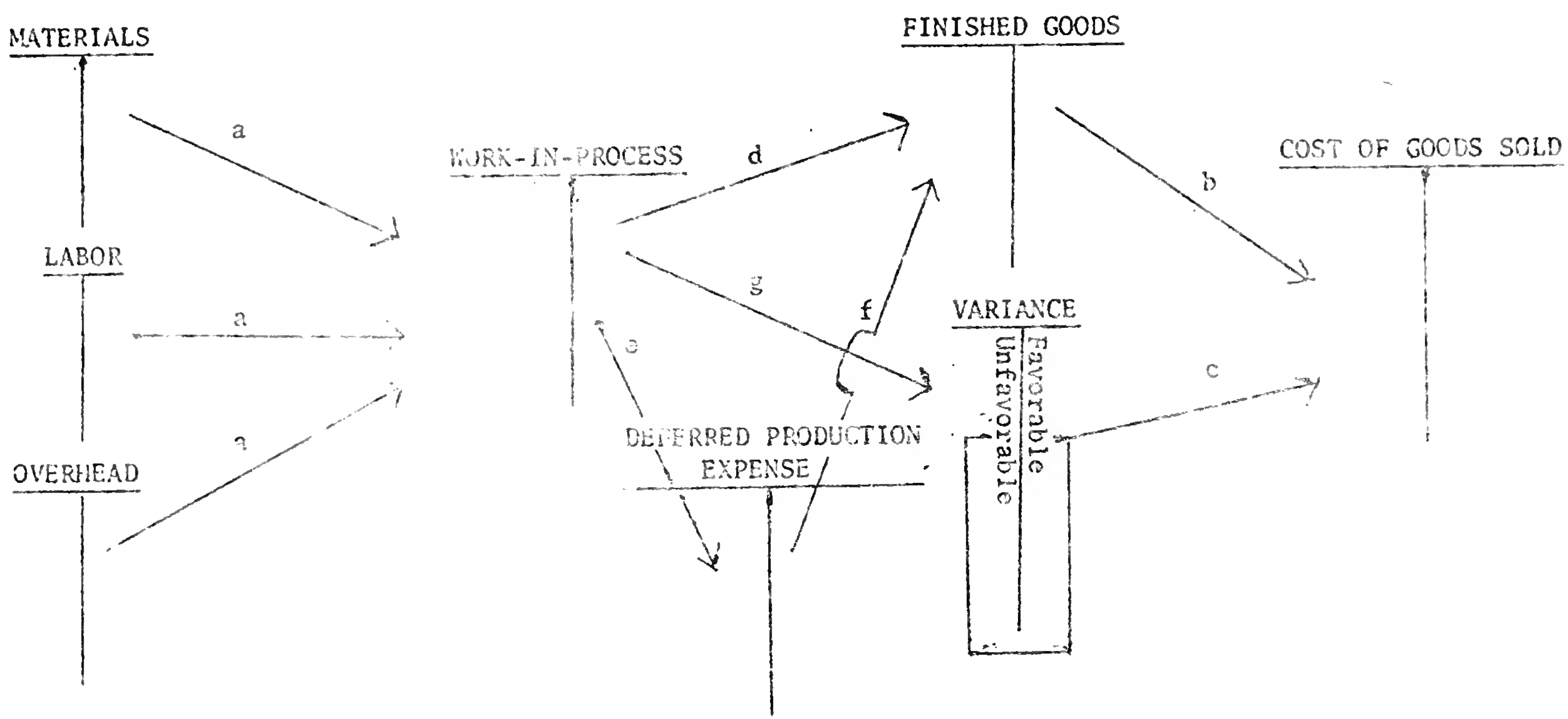
If the L-C cost allocation model was used to prepare the 19x1 and the forecasted 19x2 income statements of the XYZ Company they would appear as follows:

	19x1 Actual[12]	19x2 Forecast
Sales	$200,000	$300,000
Cost of Goods Sold	184,412	276,618
Net Income	$ 15,588	$ 23,382

In the above example, the major advantage of the use of the L-C cost allocation model over matching actual production costs with revenues is that in 19x1, when XYZ undertook a production venture that management determined would be profitable the L-C cost allocation model reported a profit while matching actual production costs with revenues reported a loss.

In satisfying himself that the use of the L-C cost allocation model is appropriate and that the model is used correctly the auditor will have to determine the answers to all of the questions previously posed about the use of learning curve models for forecasting. Additionally, he will have to satisfy himself about the reasonableness of management's estimate of the number of units to be produced. Fortunately, the results obtained with the use of the L-C cost allocation model are not changed materially by relatively large errors in the initial estimate of the number of units of

FIGURE 2
L-C Cost Allocation Model: Diagram of Flow of Costs



a. Actual costs.
b. Period sold.
c. Period incurred.
d. Projected unit cost or expected average unit cost of all anticipated production, whichever is lower
e. Excess of projected unit cost over expected average unit cost of all anticipated production.
f. Excess of expected average unit cost of all anticipated production over projected unit cost.
g. Actual less projected unit cost.

anticipated production.[13] Hence, the advantages of using the L-C cost allocation model can still be obtained even if the number of units of anticipated production are intentionally understated in an attempt to be "conservative" in income recognition.

Human Resource Accounting

One of the topics receiving considerable attention in the accounting literature in recent years is "human resource accounting." The ultimate objective of human resource accounting appears to be the determination of the value of the human resources employed in an organization. While the realization of that objective is still a long way off, more traditional accounting methods, based on historical cost, can be used today to record and amortize an organization's investment in its employees. The learning curve cost allocation model described previously can be used to record and amortize one type of investment in human resources.

When the production process follows the learning curve phenomenon, the ratio of incurred production costs to units produced declines as the number of units produced increases. Here, the production process has two joint products, one tangible, the other intangible. The tangible product is the unit produced. The intangible product is the ability to produce additional units with a lower expenditure of time and materials. This "know-how" is a job-specific human asset. It is of value because it can reduce subsequent production costs. The value of this intangible asset increases rapidly at first as managerial and production personnel quickly acquire "know-how." As production continues and employees become more

efficient, the rate of investment declines until little or no additional investment takes place. Finally, as the product's life cycle nears it end, the value of this job-specific human asset declines as the cost reduction potential of this particular "know-how" declines.

All that is necessary to account for this human resource is to use the L-C cost allocation model and relabel the "Deferred Production Expense Account" as a "Job-Specific Human Asset." It should be noted that this particular human resource account does not refer to any particular individual or small group of individuals within the organization. It refers to all employees of the organization who have anything to do with the production of the specific product in question. Like the learning curve phenomenon, it considers organizational "learning" as well as individual learning.

SUMMARY & CONCLUSIONS

To date the use of learning curves in financial accounting has been very limited. However, they appear to have significant potential applications in forecasting, cost allocation, and human resource accounting. These three uses are related. Learning curves can be used for cost allocation, human resource accounting, and forecasting simultaneously. All that is necessary is to recognize that the early period production costs deferred to later periods are in fact a form of human asset.

In evaluating forecasts and cost allocations made with a learning curve model, the auditor must satisfy himself that the use of such a model is appropriate and correct. To do this, he must be familiar with these models and their underlying assumptions and limitations.

FOOTNOTES

[1]F. J. Andres, "The Learning Curve as a Production Tool," Harvard Business Review (January-February, 1954), p. 88.

[2]F. J. Montgomery, "Increased Productivity in the Construction of Liberty Vessels," Monthly Labor Review (November, 1943), p. 861.

[3]R. B. Jordan, "Learning How to Use the Learning Curve," N.A.A. Bulletin (January, 1958), p. 27.

[4]The necessary programs are frequently available through time sharing computer service centers. See also W. J. Morse, "BASIC Programs for Implementing Learning Curve Cost Allocation Model," (unpublished paper, University of Illinois, 1973).

[5]Jordan, p. 27.

[6]For a discussion of methods to handle these complications in selected circumstances see W. J. Morse, The Allocation of Production Costs With the Use of Learning Curves (unpublished Ph.D. dissertation, Michigan State University, 1971).

[7]For a discussion of this problem and how to overcome it see: W. J. Morse, "Learning Curve Cost Projections With Constant Unit Costs," forthcoming in Managerial Planning.

[8]In the aircraft industry learning curve models are used to compute period costs and revenues in a manner similar to the precentage of completion method when the total number of units to be produced is certain.

[9]The basic data in this example is taken from W. J. Morse, "Reporting Production Costs That Follow the Learning Curve Phenomenon," The Accounting Review (October, 1972), pp. 766-767.

[10]See Appendix I for an outline of the procedures followed to compute the forecast.

[11]W. J. Morse, "Reporting Production Costs That Follow the Learning Curve Phenomenon," pp. 761-772.

[12]See Appendix II for an outline of the procedures used to compute the 19x1 income statement.

[13]W. J. Morse, "Reporting Production Costs That Follow the Learning Curve Phenomenon," p. 771.

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APPENDIX I

Outline of Forecast Computation Procedures

1) Apply least-squares regression analysis to the logarithms of the data in Table 2 to determine the parameters of the learning curve model, (a = 1, 001.23 ; b = .235308).

2) Estimate the number of units that can be produced in 19x2:

   i) Estimate total production hours available from 1/1/x1 through 12/31/x2. To avoid problems caused by rounding assume 9,895.1 hours were used in 19x1, and 10,044.8 hours are estimated to be available in 19x2 for a total of 19,939.9 hours.

   ii) Estimate the number of units that can be produced through 19x2. Using equation 8, with T = 19,939.9 hours, X = 50.

   iii) The difference between the number of units that can be produced through the end of 19x2, and the number of units that were produced in 19x1 is the estimated production for 19x2, (50-20=30).

3) Estimate total production costs for 19x2:

   i) Multiply the estimated total production hours available in 19x2 by the sum of the direct labor and variable factory overhead costs per hour, (10,044.8 x (\$5 + \$5) = \$100,448).

   ii) Multiply the direct materials cost per unit by the estimated number of units that can be produced in 19x2, (\$6,000 x 30 = \$180,000).

   iii) Sum these costs, (\$100,448 + 180,000 = \$280,448).

4) Estimate the sales revenue of 19x2 by multiplying the unit selling price by the number of units of estimated production, (\$10,000 x 30 = \$300,000).

APPENDIX II

Outline of L-C Cost Allocation Computation Procedures

1) Apply least-squares regression analysis to the logarithms of the data in Table 2 to determine the parameters of the learning curve model, (a = 1,001.23; b =.235308).

2) Obtain an estimate of the total number of units of anticipated production. Assume the estimate is 124 units.

3) Compute the estimated average unit cost of all anticipated production.

   i) Find the estimated average production time by solving for Y in equation 1 with X equal to 124 units, (Y = 322.059 hours).

   ii) Multiply the average production time by the sum of the direct labor and variable factory overhead costs per hour, (322.059 x ($5 + $5) = $3,220.59).

   iii) Add the direct materials cost per unit, ($3,220.59 + 6,000.00 = $9,220.59).

4) Determine the average cost of 20 units, ($9.220.59 x 20 = $184,412 rounding). This is 19x1's cost of goods sold.

5) Any difference between the model based cost of the first 20 units and the actual cost of the first 20 units is written off as a variance.

   i) Actual cost of first 20 units is $218,950.

   ii) Model based cost of the first 20 units is $218.950, (Solving for T

in equation 8 with X equal to 20, gives a T of 9,895.1 hours. Model based total costs are (9,895.1 x ($5+$5)) + ($6,000 x 20) = $218,950 rounding.).

iii) Hence, there is no cost variance.

6) Any difference between the model based cost of the units produced and the estimated average cost of this number of units is debited or credited to the Deferred Production Expense Account. In this case the Deferred Production Expense account is debited for $34,538, ($18,950 - 184,412).